Effective Bogomolov in Large Non-Abelian Extensions

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A field is called Bogomolov if there is a lower bound for the height of elements of infinite order. This property has been studied for almost a century and the case of a special type of non-abelian extensions is my current research topic. We take an elliptic curve E over $\mathbb{Q}$ and an infinite Galois extension K of $\mathbb{Q}$ with bounded local degrees. Then we can show that the field $K(E_{\mathrm{tor}})$ where we adjoin to K all the coordinates of torsion points of E has the Bogomolov property. The main components of the proof are a supersingular prime p, the Frobenius automorphism and many Galois diagrams. The result generalizes a result of Habegger [1] where he proved the same result for $\mathbb{Q}(E_{\mathrm{tor}})$ where E is defined over $\mathbb{Q}$. The big change here is that the field K is allowed to be an infinite extension.

References

[1] P. Habegger. Small height and infinite nonabelian extensions. *Duke Math. J.*, 162(11):2027–2076, 2013.