

6. JORDAN NORMAL FORM

$$A \text{ is in Jordan normal form} \Leftrightarrow A = \begin{pmatrix} J_1 & & 0 \\ & \ddots & \\ 0 & & J_n \end{pmatrix}$$

$$J_i = \begin{pmatrix} \lambda_i & 1 & & 0 \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{pmatrix}$$

$$B \cdot (v_1 \dots v_d) = (v_1 \dots v_d) \cdot \begin{pmatrix} \lambda & 1 & & 0 \\ & \lambda & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda \end{pmatrix} \quad (S^{-1} \cdot B \cdot S = J)$$

$$v_1 \in \text{Eig}(B, \lambda)$$

$$B \cdot v_2 = \lambda \cdot v_2 + v_1 \Leftrightarrow (B - \lambda \cdot E) \cdot v_2 = v_1 \Rightarrow \text{compute } v_i$$

$$\vdots$$

$$B \cdot v_n = \lambda \cdot v_n + v_{n-1}$$

$$\vdots$$

$$B \cdot v_d = \lambda \cdot v_d + v_{d-1} \Leftrightarrow (B - \lambda \cdot E) \cdot v_d = v_{d-1}$$

write them into columns
of base change matrix S

$$S^{-1} \cdot B \cdot S = J.$$